Nonlinear analysis of cartilage in unconfined ramp compression using a fibril reinforced poroelastic model

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Abstract

Objective. To develop a biomechanical model for cartilage which is capable of capturing experimentally observed nonlinear behaviours of cartilage and to investigate effects of collagen fibril reinforcement in cartilage.

Design. A sequence of 10 or 20 steps of ramp compression/relaxation applied to cartilage disks in uniaxial unconfined geometry is simulated for comparison with experimental data.

Background. Mechanical behaviours of cartilage, such as the compression-offset dependent stiffening of the transient response and the strong relaxation component, have been previously difficult to describe using the biphasic model in unconfined compression.

Methods. Cartilage is modelled as a fluid-saturated solid reinforced by an elastic fibrillar network. The latter, mainly representing collagen fibrils, is considered as a distinct constituent embedded in a biphasic component made up mainly of proteoglycan macromolecules and a fluid carrying mobile ions. The Young's modulus of the fibrillar network is taken to vary linearly with its tensile strain but to be zero for compression. Numerical computations are carried out using a finite element procedure, for which the fibrillar network is discretized into a system of spring elements.

Results. The nonlinear fibril reinforced poroelastic model is capable of describing the strong relaxation behaviour and compression-offset dependent stiffening of cartilage in unconfined compression. Computational results are also presented to demonstrate unique features of the model, e.g. the matrix stress in the radial direction is changed from tensile to compressive due to presence of distinct fibrils in the model.

Relevance

Experimentally observed nonlinear behaviours of cartilage are successfully simulated, and the roles of collagen fibrils are distinguished by using the proposed model. Thus this study may lead to a better understanding of physiological responses of individual constituents of cartilage to external loads, and of the roles of mechanical loading in cartilage remodelling and pathology. © 1999 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Articular cartilage consists of a collagen fibrillar network entrapping a noncollagenous matrix mainly made up of proteoglycan macromolecules swollen by a fluid carrying a variety of mobile ions. The fraction of fluid content generally varies with the depth from the articular surface. In healthy adult cartilage, the fluid content can make up to as much as 85% of the total mass by wet weight in the most superficial 25% of the cartilage, and decreases to about 70% at the subchondral bone [1]. The collagen (mainly type II with some type VI, IX, XI) accounts for approximately half of the tissue mass by dry weight [2]. The proteoglycans account for 20–30% of the total dry weight of the tissue in healthy adult cartilage and the fractional content increases with depth. The collagen fibrils are strong in tension whereas the stiffness for compression is expected to be relatively lower due to their slenderness. The proteoglycan matrix predominately supports compression due to its propensity to expand, an effect produced by a high concentration of fixed negative charged groups. The predominant role of proteoglycan associated electrostatic forces in cartilage compressive stiffness has been confirmed by both experiments [3] and models.
[4], although a recent study also suggested a role of collagen in compressive strains [5].

In short, cartilage is a highly nonhomogeneous, anisotropic and multiphase biomaterial. However, in order to avoid difficulties in mathematical modelling and computation, the homogeneous isotropic biphasic model [6] has been widely employed. The model has been considered to successfully describe experimental data of confined compression [2,6], although recent experimental and model studies have suggested some limitations to the interpretation of confined compression tests using this model, due to difficulty to quantitate interdigitating boundary conditions between the cartilage and the porous filter and the unknown level of confinement of the specimen in its testing well [7]. In the case of unconfined compression with frictionless platen/cartilage interfaces, it has been known that the isotropic homogeneous model cannot account for the large stress relaxation transients and can at most predict a ratio of 3:2 for the peak to equilibrium loads while experiments can show 5-10:1 ratios [8]. This ratio of peak to equilibrium loads can be increased by appealing to friction at the cartilage/platen interfaces, however, in this case it still falls short of experimental values [9,10]. The model has also encountered difficulty in explaining creep indentation tests for short time behaviours [11]. Cohen et al. [12,13] considered the material to be transversely isotropic and the model they proposed was able to fit data from creep indentation and unconfined compression experiments. Like the isotropic model, this transversely isotropic model considers the drained cartilage, the solid components, also as a single constituent, i.e. the distinct individual roles of matrix (mainly proteoglycans) and fibrils (mainly collagen) are not distinguished. Another approach separates the solid phase into at least two constituents which possess distinct physicochemical properties and which perform particular physiological functions, as suggested in Bass et al. [14].

Recently Soulihat et al. [15] suggested a model which formally presents an elastic collagen fibrillar network which resists tension only, reinforcing an isotropic biphasic component representing proteoglycans and water. A linear formulation was proposed and the corresponding analytical solution was found for uniaxial unconfined compression. In the present work, the model is further developed to include three nonlinear features, i.e. the dependence of permeability on the dilatation of the material, the effect of finite deformation and the fibril stiffening with its strain in the longitudinal direction, which is hypothesized to be the most significant nonlinear factor in cartilage mechanical behaviour. The drained isotropic matrix constituent is considered as linearly elastic. General behaviour of the model along with its distinguishing features and its success in describing an experimentally known nonlinear response of cartilage in unconfined compression are presented and discussed.

2. A fibril reinforced poroelastic model

Consider a cylindrical sample of cartilage with thickness h and radius R. The cartilage is modelled as an isotropic solid matrix containing fluid-saturated pores entrapped by a fibrillar network (Fig. 1). The isotropic matrix with pore fluid is considered as poroelastic, where both the solid and the fluid phases are assumed to be incompressible. Three material properties are required to define the isotropic biphasic component: the Young’s modulus $E_m$ and Poisson’s ratio $\nu_m$ of the drained matrix and the hydraulic permeability $k$. Other necessary assumptions are stated as follows. The hydraulic permeability depends on the dilatation of the bulk material. The fibrils are evenly distributed in the radial, circumferential and axial directions (i.e. r-, $\theta$- and z-directions), forming an elastic constituent attached to the porous matrix. The Young’s modulus of the fibrillar network is $E_f$ in any of the three orthogonal directions and depends on the longitudinal strain of the fibrils. These fibrils have no resistance to compression, i.e. the Young’s modulus for compression is assumed to be zero. The effect of lateral deformation of every single fibril is also neglected, i.e. the only fibril stress considered is the tensile stress produced by stretching in the longitudinal direction. This is a reasonable approximation given the slenderness of the collagen fibrils and their high stiffness in tension. Thus the fibrils employed in the model are one dimensional nonlinear structural elements which do not support compression. An elastic theory applies to the fibril constituent while a poroelastic theory is employed for the biphasic component.

Two types of material nonlinearity are considered, i.e. the dependence of the permeability on the dilatation and

![Fibrillar network (collagen)](image)

![Porous matrix (proteoglycan)](image)

Fig. 1. Diagram of the model showing the isotropic matrix (containing pores) and the fibrils evenly distributed in the three orthogonal directions. Four properties are required to define the material: the Young’s modulus and Poisson’s ratio of the drained matrix $E_m$ and $\nu_m$, the Young’s modulus of the fibrillar network $E_f$ and the permeability of the bulk material $k$. Furthermore, $E_f$ and $k$ are strain dependent: $E_f = E_f|_0 + \hat{E}_f|_0 \hat{e}_f$, where $\hat{e}_f$ is the tensile fibril strain ($E_f|_0 = 0$ if $\hat{e}_f$ is compressive) and $k = k_0 \exp(M\epsilon)$, where $\epsilon$ is the solid dilatation. Thus the nonlinear model has six material parameters, $E_m$, $\nu_m$, $E_f$, $\hat{E}_f|_0$, $k_0$ and $M$. **
the fibril stiffening with its tensile strain. In the present investigation, the permeability variation is taken to be
\[ k = k_0 \exp(Me), \]
which was proposed in Lai and Mow [16]. Here \( \varepsilon \) is the dilatation of the bulk material and \( k_0 \) is the permeability of the reference state. Nonlinear constitutive behaviour of cartilage in tension was reported in literature (e.g. Pins et al. [17], Roth and Mow [18]). Pins et al. [17] found a nonlinear variation of Young’s modulus of uncrosslinked collagen fibre with its nominal strain, showing a Young’s modulus varying from 1.82 MPa at 0% strain to 45 MPa at 50% strain. In the current model, the nonlinear behaviour of cartilage in tensile stiffness is captured by the dependence of the fibril network modulus on the tensile strain. A linear variation is proposed for the tangent Young’s modulus of the fibrillar network,
\[ E_t = E_t^0e + E_t^0, \]
where \( e \) is the tensile strain (longitudinal) of the fibrils, and \( E_t^0 \) and \( E_t^0 \) are constants (when \( e \) is compressive, \( E_t = 0 \)). These nonlinear parameterizations of \( k \) and \( E_t \) are also motivated by the results of fitting experimental data from unconfined compression to the analytical, linear fibril reinforced model [19]. In that study, increasing compression offset appeared to reduce \( k \) and increase \( E_t \), as approximated by fitting the linear model to experimental stress-relaxation profiles.

Analytical solutions are not expected for such highly nonlinear problems, which also include the effect of finite deformation. Thus we seek numerical solutions using the commercial finite element code ABAQUS 5.5 (Hibbit, Karlsson and Sorensen, USA). A porous medium is modelled in ABAQUS by the conventional approach as a multi-phase material adopting an effective stress (the stress on solid matrix excluding the pore pressure) principle to describe its behaviour. The consolidation theory based on this principle has been widely accepted for study of mechanical behaviours of cartilage, e.g. Broom and Oloyede [20,21]. We use one specific case of the formulation in ABAQUS in which pores are saturated with only one fluid (a liquid; ‘saturation’ = 1), temperature is not involved, and both the solid and liquid phases are intrinsically incompressible. The stress for the biphasic porous material including matrix and fluid but not fibrillar network takes the form
\[ \sigma_{\text{porous}} = \sigma^m - p, \]
where the effective stress \( \sigma^m \) (which will be simply referred to as the matrix stress hereafter) and the pore pressure \( p \) are formulated in ABAQUS. The total stress on the overall material is
\[ \sigma = \sigma_{\text{porous}} + \sigma^f, \]
after the stress on the fibrillar network \( \sigma^f \) is included.

Forchheimer’s law is employed in ABAQUS to describe the fluid flow for which the actual permeability is dependent on the fluid velocity relative to the solid. This dependency is neglected in the present investigation due to low fluid velocity occurring in cartilage and thus Darcy’s law is adopted, i.e. the fluid flow rate relative to the solid is proportional to the fluid pressure gradient.

In a finite deformation, true stresses (Cauchy stresses) are calculated. For our situation in which the principal strain directions coincide with the directions of the coordinate axes, the strain adopted is simply equivalent to the logarithmic strain, which for the case of one-dimensional uniform strain is given as
\[ \varepsilon = \ln(L/L_0), \]
where \( L \) is the time dependent length with initial value \( L_0 \). The difference between logarithmic and nominal strains can be observed from the following example. If a disk with thickness 1 mm is compressed by 0.1 mm, the nominal and logarithmic strains in the direction of compression are −10% and −10.54%, respectively. The differences produced by using different strain measures are much less significant in the lateral direction, since the maximum strain expected in this direction is less than 1%.

In order to incorporate Eq. (1) into ABAQUS, the dilatation is represented by the current void ratio \( e \) (pore volume over solid volume) and its initial value \( e_0 \) (\( e_0 = 4 \) in computations), resulting in the following relation in finite deformation [22].
\[ k = k_0 \exp \left( M\frac{e - e_0}{1 + e_0} \right). \]

3. Finite element simulation

Uniaxial unconfined compression tests were simulated by a finite element procedure. In these tests, a cylindrical specimen is placed between two rigid impermeable platens in a chamber, where the specimen is immersed in a fluid (for example, PBS, 0.15 M NaCl, pH = 7.4). The surfaces between the specimen and the platens are taken to be frictionless so that the specimen can expand freely in the lateral (radial) direction while compression is applied in the axial direction. The fluid can exude out of the cartilage via the free cylindrical surface [19].

This problem is axisymmetric. Furthermore, all field variables are independent of \( z \) except the vertical displacement which varies linearly in \( z \). The upper half of the disk is discretized. The mesh is shown in Fig. 2, where \( z = h/2 \) refers to the top surface of the specimen. It turns out that such a fine mesh (\( \beta_1 = 0.5 \) and \( \beta_2 = 0.25 \)) is not always necessary. For example, a division of three or four elements in the vertical direction is sufficient when the top surface is not adhesive.

The biphasic porous medium is modelled by 8-node axisymmetric elements including pore pressure, i.e. the element CAX8P or CAX8RP in ABAQUS. The two
element types are identical except the later adopts a reduced integration technique; no visible differences in the results were noted between the two elements when used in our cases. The displacements are interpolated at all nodes and are thus quadratic. The pore pressure is interpolated at the four corner nodes only and thus gives a bilinear variation within an element. On the other hand, the fibrils are represented by a system of special springs which resist tension only (SPRINGA in ABAQUS). Vertical springs are not necessarily presented when the contacts are nonadhesive, since they will undergo compression and thus develop no stress (prestresses are not considered due to lack of such data). The total stiffness of the horizontal fibrils within one continuum element domain can be determined using the method described in Appendix A. This total stiffness is subsequently distributed between one longer and four shorter springs (Fig. 2). It is assumed that the longer spring shares 2/3 of the total stiffness in order to guarantee a uniform stress state when a uniform strain is produced. The stiffness of the vertical springs can be determined in a similar way.

The boundary conditions are as follows: at \( z = h/2 \), a given vertical displacement is applied which varies with time and the surface is impermeable; at \( z = 0 \), the same conditions apply as at \( z = h/2 \) but the given vertical displacement is zero; at \( r = R \), the surface is permeable; at the \( z \) axis, symmetry implies no fluid flow and no solid displacement in the horizontal direction. An impermeable boundary is a natural boundary condition in the formulation in ABAQUS and thus requires no input.

4. Results and discussion

All three nonlinear factors, i.e. the fibril stiffening with its tensile strain, the effect of finite deformation and the dependence of permeability on the dilatation, are considered in all numerical computations except for the cases specified in Fig. 6.

Initially, we verify whether this procedure can simulate the so-called Mandel–Cryer effect which has been used as a benchmark problem for testing the validity of numerical codes of poroelasticity [23]. Mandel [24] considered an infinitely long rectangular specimen of isotropic poroelastic material sandwiched at the top and bottom by two rigid frictionless impervious plates. The lateral sides of the specimen are free and permeable. If compressive forces are suddenly applied to the plates (a Heaviside loading in force), the pore pressure at the centre of the specimen rises beyond its initial value following Heaviside loading and then decays to zero after reaching a maximum. This is so because as the fluid drains out of the specimen via the edges, a larger portion of the applied load is transferred toward the effectively stiffer central region of the specimen (the stiffness depends on the pore pressure). This effect was also observed by Cryer [25] for a spherical specimen under hydrostatic pressure. Li et al. [26,27] found similar behaviours in light structural elements as beams and plates.
with diffusion in the longitudinal directions, not only for pore pressure but also for stresses and displacements. An extensive discussion on these behaviours was presented in Cederbaum et al. [28] For the current problem, a sequence of 10 steps of equal displacement increments is applied as stated earlier (Fig. 3) but every displacement increment is imposed in no time instead of 5 s. The variation of load (which is equal to the resultant of $\sigma_z$ over a horizontal section) against time is shown in Fig. 4a. The pore pressure $p$ and the total stress $\sigma_z$ at the centre ($r = 0$) are shown for the first 40 s of the 8th step (Fig. 4b). The Mandel–Cryer effect is observed even for our relaxation case, i.e. $p$ and $\sigma_z$ at the centre rise for some time when the integrated load decreases sharply. We would see a sharper rise in $p$ and $\sigma_z$ at the centre in a creep test when the load remains constant.

In order to see if the current model can simulate experiments, and to examine further the numerical procedure, four sets of experimental data [19] labelled A, B, C and D are used for comparison. These data of force vs time were directly taken from four independent tests. The radius $R$ was 1.40 mm for all four tests; the thickness $h$ was 1.11, 1.02, 1.02 and 1.065 mm, respectively for the tests A, B, C and D. The required material properties are estimated from the data including the peak loads at short times when mechanical diffusion begins, the loads at equilibrium when fluid flow ceases and the relaxation time in each step, as well as the lateral deformation. The latter one is not available for the four tests, thus the Poisson’s ratio is assumed with reference to Jurvelin et al. [29] In particular we take $\nu_m = 0.42$ so that the time dependent effective Poisson’s ratio $\nu_{\text{eff}}$ (i.e. the ratio of tensile lateral strain to compressive axial strain for the current time) obtained from computation is principally consistent with those found in Jurvelin et al. [29] Either a smaller $\nu_m$ with a larger $E_m$ or a larger $\nu_m$ with a smaller $E_m$ can also possibly fit the experimental load variations. For the current model, $\nu_{\text{eff}}$ is dependent on the mechanical diffusion process and on the axial strain since the fibrils are hardening when the strain grows. When $\nu_m$ is taken to be 0.42, $\nu_{\text{eff}}$ is approximately 0.16 and 0.05, respectively for the 1st step and 20th step at equilibrium; $\nu_{\text{eff}} > 0.15$ for the first six steps and $\nu_{\text{eff}} > 0.10$ for the first 12 steps at peaks (Test C and D). If $\nu_m$ is smaller, the change of $\nu_{\text{eff}}$ with respect to time would be smaller (this is also true for a smaller $E_f$). The parameters used in computation for the purpose of comparison with experimental data are given in Table 1. These parameters do not represent unique best-fit values, but are found to reasonably match experimental data.

A comparison between experimental data and model simulations demonstrates the ability of the model to capture: (1), the strong relaxation behaviour with no theoretical limit for the ratio of the peak load to the load at equilibrium and (2), the nonlinear compression-offset dependent stiffening of the transient response, where relative peak stresses with respect to equilibrium increase at later steps in the ramp sequence. Reasonable agreement between the experimental (cases A and C) and computational results is observed (Fig. 5). Similar agreement is also found for the other two cases which are not shown.

A reference case is now considered for which the material parameters, shown in the last row of Table 1, are taken to represent average behaviour found in Tests A–D. In addition, we take $R = 1.40$ and $h = 1.00$ mm. It was previously shown [15] that the ability of the present model to simulate the large ratio of the load at peak over the load at equilibrium is due to presence of fibrils which resist tension only. We will further show that the ability to simulate the compression-offset dependent stiffening is predominantly due to the inclusion of the material nonlinearity involving fibril stiffening with its tensile strain. Firstly we take fibrils to be linear, with $E_f = 6$ so that the first peak load is at the same level for all cases.
When none of the three nonlinearities is considered, the relative peak loads with respect to equilibrium loads (i.e. $\Delta F = F_{\text{peak}} - F_{\text{equilibrium}}$ for each step) are identical for all steps (Fig. 6a, linear case). When either the nonlinearity of permeability (NL permeability) or the effect of finite deformation (FD) is taken into account, $\Delta F$ grows with compression offset. However, the variation is small compared to

<table>
<thead>
<tr>
<th>Cases considered for $E_m$ (MPa) and $v_m$</th>
<th>Young's modulus of fibrillar network (MPa) $E_i = E_i^0 + E_i^1$</th>
<th>Permeability ($10^{-15}$m$^4$/Ns) $k = k_0 \exp(M \epsilon)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test A</td>
<td>0.262, 0.42</td>
<td>2880, 3</td>
</tr>
<tr>
<td>Test B</td>
<td>0.238, 0.42</td>
<td>1200, 3</td>
</tr>
<tr>
<td>Test C</td>
<td>0.290, 0.42</td>
<td>2420, 3</td>
</tr>
<tr>
<td>Test D</td>
<td>0.250, 0.42</td>
<td>2800, 1</td>
</tr>
<tr>
<td>Reference</td>
<td>0.260, 0.42</td>
<td>2400, 3</td>
</tr>
</tbody>
</table>

Table 1: Material parameters used in finite element simulations

Fig. 5. Simulation of experimental results using the proposed fibril reinforced model: (a), the curve for load vs time for Test A is shown, which is almost overlapped with the one obtained from computation using the parameters shown in Table 1; (b), the curve for load vs. time for Test C is shown together with the one from computation using the parameters shown in Table 1. In each case the experimental data are simply taken from a single test and no data processing is done.

Fig. 6. Load vs time for which individual nonlinear factors are considered. Parameters are $R = 1.4$, $h = 1$, $E_m = 0.26$, $v_m = 0.42$ for all cases and $E_i = 6$ when the fibrils are considered as linear. Additional curves are drawn to clearly show the values of the peak loads for each case (as will be done later). (a), for linear solution (lower curve, $k = 3$), for nonlinear permeability only (middle, $k_0 = 3$ and $M = 30$) and for finite deformation only (upper, $k = 3$); the differences for earlier steps are smaller and are not shown in order to present a clear figure for the later steps; (b), for the reference case when three nonlinearities are involved (parameters shown in Table 1) and for the case when the fibrils are considered as linear while other two nonlinearities are included.
experimental observations, as far as the current parameters and compression offset are concerned. Even when these two nonlinearities (FD + NL permeability, Fig. 6b) are involved at the same time, the change over the step sequence is still much smaller than that in the reference case which accounts for the fibril nonlinearity as well. This suggests that the fibril stiffening produces the large differences in $\Delta F$ over steps (i.e. $\Delta F$ grows quickly with successive steps). Thus it is only with the addition of fibril stiffening with its tensile strain that the experimentally observed compression-offset dependent stiffening of the transient response can adequately be simulated (Reference case, Fig. 6b).

In order to perform parametric studies, twelve additional cases are calculated based on the reference case. These cases are designed so that for each case five of the six parameters are left identical to the reference values while the remaining one is altered by $\pm 5\%$, $10\%$ or $20\%$ of its corresponding reference value. It is observed that a change in $E_m$ or $v_m$ mainly changes the stiffness of the disk at equilibrium but produces almost no change in $\Delta F$ (Fig. 7). A bigger change in stiffness at equilibrium is observed when $v_m$ is raised or lowered by $5\%$, as compared with the situation of increasing or reducing $E_m$ by $10\%$. Furthermore increasing $v_m$ by $5\%$ produces a greater change in load than reducing it by $5\%$.

A reduction in permeability could increase the peak values and extend the relaxation time. However for the current reference value of $k_0$, a change of $20\%$ does not change the peak values significantly (not shown). On the other hand, a change in $M$ significantly alters the relaxation time for the later steps (Fig. 8a), since the permeability $k$ is changed significantly when compression is large ($k = k_0 \exp(M\epsilon)$). For the current model, the large peak loads are controlled by the fibrils; a change in $E^f$ (of the Young's modulus $E_f$) mainly alters the peak load values but has little effect on the load at equilibrium (Fig. 8b). For the material considered, $E^f_0$ is rather small compared to $E_f$. For the reference case ($E^f_0 = 3$), $E_f = 4.211, 7.473$ and $14.30$ at equilibrium for

![Fig. 7. Load vs time for changes in $E_m$ and $v_m$; in the middle of each figure is the reference curve; the upper and lower curves correspond to bigger or smaller parameters with respect to the reference case. (figure a: $E_m \pm 10\% E_m$; figure b: $v_m \pm 5\% v_m$). The parameters for the reference are $E_m = 0.26$, $v_m = 0.42$, $E^f_0 = 2400$, $E^f_1 = 3$, $k_0 = 3$ and $M = 30$.](image)

![Fig. 8. Load vs time for changes in $M$ and $E^f_1$: (a), $M$ is respectively raised and reduced by $20\%$ with respect to the reference case; (b), $E^f_1$ is, respectively raised and reduced by $20\%$ with respect to the reference case. Only steps 11–20 are shown; deviations for earlier steps are smaller.](image)
the 1st, 5th and 20th steps respectively; while at the peaks $E_f = 8.949$, 12.85 and 20.61 for the 1st, 5th and 20th steps due to fibril hardening during the ramp (we see that $E_f$ increases proportionately more at earlier steps than at later steps). Hence a change in $E_f^i$ influences the results mainly at the earlier steps (not shown).

It is interesting to investigate the stresses in individual phases to demonstrate some other unique features of the fibril reinforced model. The porous stresses, i.e. the stress in both matrix and fluid phases not including the fibril stress, are shown in Fig. 9a for axial and radial directions ($\sigma_z = \sigma_z^m - p$ and $\sigma_r^{\text{porous}} = \sigma_r^m - p$; the total stress $\sigma_z = \sigma_z^{\text{porous}}$ due to no fibril stress in the axial direction), together with the stress of the fibrillar network $\sigma_r^f$. Both porous stresses are compressive. The relative peak values for $\sigma_z$ and $\sigma_r^{\text{porous}}$ with respect to their corresponding values at equilibrium are of the same order of magnitudes for the same step, since they are mainly contributed by the pore pressure (at short times). The transient stress of the fibrillar network (Cauchy stress) is determined by

$$\sigma_r^f = \frac{(0.5E_r^e(E_f^i + E_f^0))e_r}{\exp(e_z)e_z},$$

i.e. the nominal stress (the numerator) modified by the area change (the denominator; which would be $(1 + e_z)(1 + e_0)$ using engineering strains). $\sigma_r^f$ is always tensile and identically balances $\sigma_r^{\text{porous}}$ at equilibrium. Should the fibrils be absent, $\sigma_r^{\text{porous}}$ would vanish at equilibrium when an uniaxial state of compression stress is reached. The fibril's function is further demonstrated by considering the matrix stresses ($\sigma_z^m$ and $\sigma_r^m$), for which neither pore pressure nor fibril stress is included. The compressive axial matrix stress $\sigma_z^m$ increases even during relaxation; and the radial matrix stress $\sigma_r^m$ increases during loading and decreases during relaxation ($\sigma_r^m$ is tensile only at the beginning of the first step).

Fig. 9. Variation of stresses against time in different phases at the centre ($r = 0$) for the reference case: (a), porous stresses in axial and radial directions ($\sigma_z = \sigma_z^m - p$ and $\sigma_r^{\text{porous}} = \sigma_r^m - p$, both are compressive) and tensile stress in the fibrillar network ($\sigma_r^f$); (b), matrix stresses in axial ($\sigma_z^m$) and radial ($\sigma_r^m$) directions for the first 60 s for which offsets are applied instantaneously and in 5 s, respectively. $\sigma_r^{\text{porous}}$ is compressive and is quite large even at equilibrium, though the strain in that direction is always tensile and so is the stress of the fibrillar network $\sigma_r^f$; the compressive axial matrix stress $\sigma_z^m$ increases even during relaxation; and the radial matrix stress $\sigma_r^m$ increases during loading and decreases during relaxation ($\sigma_r^m$ is tensile only at the beginning of the first step). (Erratum: unit for Fig.b should be 'kPa')

![Fig. 9](image-url)

Fig. 10. Patterns for pore pressure: (a), and radial strain; (b), for different times for the reference case at the 20th step, where the time $t$ appeared in the legends is counted from the beginning of the step. The pore pressure at the central area continues increasing for some time after the compression offset reaches its maximum value at $t = 5$ (Mandel-Cryer effect); the pore pressure near the edge decays rapidly in a monotonic manner. In the central area the strain changes little with position.

![Fig. 10](image-url)
Fig. 9b). This is so since part of the load which was supported by the fluid is being transferred to the matrix when mechanical diffusion proceeds. Furthermore, a unique feature of the model is observed while studying temporal variation of $\sigma_m^r$: the radial stress rises when an offset in the axial direction is being applied and then decreases to become compressive towards equilibrium (Fig. 9b). This is true for every step but only the first step is shown. The radial strain $\varepsilon_r$ is increased when the disk is being compressed further, resulting in an increase in $\sigma_m^r$. Although $\varepsilon_r$ is always tensile, it decreases during relaxation, resulting in a negative increment in $\sigma_m^r$. Thus the drained matrix (fibrils not included) is in tension in the radial direction at the very beginning but in compression later on. Should the fibrils be absent, the drained matrix is always in tension in the radial direction ($\sigma_m^r = 0$ at equilibrium). Obviously the phenomenon is due to existence of two distinct solid constituents of the tissue and can be observed at any internal position; it is not related to the Mandel–Cryer effect [24,25], which results from the quick redistribution of the pore pressure in a local domain (i.e. around $r = 0$ in this case).

Finally, distributions of pore pressure and radial strain along the radius are presented (Fig. 10). The pore pressure varies in a spatially and temporally specific manner, demonstrating the Mandel–Cryer effect in the central domain. The gradient of the strain is very large near the edge and zero at the centre. This variation is attenuated with time to equilibrium.

5. Conclusion

A nonlinear fibril reinforced poroelastic model has been proposed for articular cartilage. The distinguishing features of the model are presented by nonlinear fibrils which support tension only and which stiffen with tensile strain. The Young’s modulus of the fibrillar network is not necessarily linearly dependent on tensile strain, although this is the case in the present work. The model and the numerical procedure have been examined parametrically and by comparing predictions to experimental data. The material parameters obtained by fitting the experimental results are compatible with those reported in the literature [8,16,18,19,29,30]. The ratio of the peak to equilibrium load for each step and its variation over a sequence of ramp compression are predominantly determined by the Young’s modulus of the fibrillar network and its strain dependency. This feature allows the model to describe experimentally observed compression-offset dependent stiffening of the transient response to ramp compression. Fibril reinforcement of the matrix can also change the quality of the matrix stress, particularly in the radial direction, from tensile to compressive. Consequences for physiological responses to load are evident.

Acknowledgements

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Appendix A. Derivation of the stiffness for the springs

The fibril network is discretized into spring elements whose stiffness is derived here. Consider a disk consisting of fibrils in $r$- and $\theta$-directions. The radius of the disk is $R$ and the thickness is $h$. When subjected to a uniformly distributed normal radial load at the circumferential boundary, a uniform stress state in the disk plane is achieved, i.e. $\sigma_r = \sigma_\theta$ and both stresses are independent of position. Thus noting that $\sigma_r = E_r(\varepsilon_r)\varepsilon_r$ (since each fibril is considered as one dimensional), we write the strain energy as

$$U_{\text{fibril}} = \int_V \left( \int_{\varepsilon_r} \sigma_r \, d\varepsilon_r + \int_{\varepsilon_\theta} \sigma_\theta \, d\varepsilon_\theta \right) \, dV = 2\pi R^2 h \int_{\varepsilon_r} E_r \varepsilon_r \, d\varepsilon_r. \quad (6)$$

Now the disk is discretized to a system of springs of length $R$. All springs are arranged in the radial direction and uniformly distributed along the circumferential direction. The springs can be considered to be fixed at the ends in the centre due to the symmetry of the problem. Loads are applied at the other ends around the circumference. Because all springs have equal length and loadings, the strain energy of the system is given by

$$U_{\text{spring}} = \int_\delta K \delta \, d\delta, \quad (7)$$

where $\delta$ is the length change of a spring and $K$ the total stiffness of all springs. Since fibrils are nonlinear, $K$ is dependent on $\delta$. For small deformations, $\varepsilon_r = \delta/R$ and thus Eq. (6) becomes

$$U_{\text{fibril}} = 2\pi R h \int_\delta E_r \delta \, d\delta. \quad (8)$$

The measures of strain energy must be identical, i.e. $U_{\text{fibril}} = U_{\text{spring}}$. Since this is true for any integration intervals, we conclude that

$$K = 2\pi h E_r. \quad (9)$$

For large deformations, the strain is defined as

$$\varepsilon_r = \ln \left( \frac{R + \delta}{R} \right) \quad (10)$$

or

$$\delta = R[\exp(\varepsilon_r) - 1]. \quad (11)$$
Equating Eqs. (6) and (7) after \( \delta \) is replaced by Eq. (11), we conclude

\[
K = \frac{2\pi h e_r}{|\exp(e_r) - 1|\exp(e_r)}.
\]  

(12)

In a finite element procedure, the springs are further discretized in the longitudinal direction. The stiffness of the shorter springs can be determined by Eq. (9) or Eq. (12) and their geometry. In the current study in which the radial strain is very small, the difference between Eqs. (9) and (12) disappears \( (e_r < 1\% \text{ if } e_z = 10\%) \).

References